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On the critical behavior of D1-brane theories

Eva Silverstein[ab] **and Yun S. Song**[b]

[a] *School of Natural Sciences, Institute for Advanced Study*
Olden Lane, Princeton, NJ 08540, USA
[b] *Department of Physics and SLAC, Stanford University*
Stanford, CA 94305/94309, USA
E-mail: evas@slac.stanford.edu, yss@leland.stanford.edu

Abstract: We study renormalization-group flow patterns in theories arising on D1-branes in various supersymmetry-breaking backgrounds. We argue that the theory of N D1-branes transverse to an orbifold space can be fine-tuned to flow to the corresponding orbifold conformal field theory in the infrared, for particular values of the couplings and theta angles which we determine using the discrete symmetries of the model. By calculating various nonplanar contributions to the scalar potential in the worldvolume theory, we show that fine-tuning is in fact required at finite N, as would be generically expected. We further comment on the presence of singular conformal field theories (such as those whose target space includes a "throat" described by an exactly solvable CFT) in the non-supersymmetric context. Throughout the analysis two applications are considered: to gauge theory/gravity duality and to linear sigma model techniques for studying worldsheet string theory.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Conformal Field Models in String Theory.

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Contents

1. Introduction

Getting a handle on the quantum behavior of non-supersymmetric systems is important in order to be able to concretely approach hierarchy problems and other aspects of supersymmetry breaking. This is also ultimately important in studying any theory, supersymmetric or not, since generic phenomena are not protected by supersymmetry.

In this paper we will analyze the quantum field theories arising on D1-branes in various supersymmetry-breaking backgrounds. We will focus mostly on D1-branes transverse to symmetric orbifold singularities, but will for comparison consider also D1-branes in combination with D5-branes and transverse to asymmetric orbifold backgrounds.

We have in mind two main motivations (or potential applications for these theories). The first concerns their role in gravity/gauge theory dualities [1, 2]. The strong coupling regime in the RG flow of the worldvolume theory of D1-branes in

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flat space has a weakly-coupled supergravity dual [3]. On a circle, this theory defines matrix string theory [4]; there the strong coupling regime of the RG flow provides a non-perturbative formulation of interacting string theory. It would be very nice to know whether this flow pattern persists in situations with less supersymmetry. In particular we would like to understand whether the regime of finite (but large) Yang-Mills gauge coupling survives, whether a mass gap develops, and so on. As in [5], one can use the gravity/gauge theory duality to translate the question of whether a string-scale vacuum energy arises into a potentially simpler question in the dual gauge theory. The second application is to linear sigma model techniques [6, 7, 8] for studying worldsheet conformal field theory via a relatively simple ultraviolet field theory that flows to the desired CFT in the infrared.

As we will review, in the large-N limit this flow pattern does persist for "quiver" gauge theories [9, 10] obtained on D1-branes at orbifold singularities [5, 11, 12]. Even for finite N, these theories have the promising feature that the target space (classical moduli space) of the theory in the ultraviolet would, if taken as the target space of a sigma model, produce a conformal field theory (here simply the corresponding orbifold conformal field theory). This is quite different from the situation obtained in studying, for example, the $\mathbb{CP}^n$ model, for which the classical moduli space is not Ricci-flat, and in which a large-N analysis reveals the presence of a mass gap [13].

Because of the promising start the quiver theories have at leading order in the $1/N$ expansion, it is worth checking explicitly whether these pleasant features persist at finite N. We find by explicit calculation that they do not for generic $(1+1)d$ quiver theories. (Similar questions in $(3+1)d$ were studied by explicit calculation in [14, 15]). We show, by way of constrast, that they do persist, at the order we compute, for the worldvolume theories arising on D1-branes transverse to asymmetric orbifolds such as [16].

We argue, using various simple features of this type of model (such as the form of the UV target space mentioned above), that the quiver gauge theories can however be fine-tuned to flow to the corresponding orbifold conformal field theory for particular choices of the couplings and θ-angles. In particular we show how to identify using the quiver theories the values of the theta angles at the orbifold conformal field theory point for $\mathbb{Z}_k$ orbifolds with $k > 2$ (as determined using wrapped D-branes in [17], giving a generalization of the result for $\mathbb{Z}_2$ orbifolds determined by Aspinwall [18]).

This statement about the flow involves a comparison of relevant operators and discrete symmetries in the two limiting theories. The situation might be compared to that of non-supersymmetric Landau-Ginzburg theories which flow to minimal model CFT's in the IR [6]. It has potential utility as an approach for studying twisted-sector tachyon condensation in perturbative string theory. The fact that there is a field theory flow between the two limiting theories raises the interesting question as to whether there is a gravitational dual to this trajectory in coupling space.

In supersymmetric contexts, one often finds subspaces on the moduli space of $2d$ conformal field theories on which the CFT becomes singular. (Examples are the points of non-perturbatively enhanced gauge symmetry in type IIA compactification on K3 [19], the conifold singularities of Calabi-Yau compactification [20, 8], and the small instanton singularities of the D1-D5 system [21].) We study a simple model (based trivially on a CFT that factorizes into a supersymmetric part times a nonsupersymmetric part) in which this persists without supersymmetry and discuss the potential implications for the spacetime string background described by this worldsheet conformal field theory.

The outline of this paper is as follows. In section 2 we introduce the theories we will study. In section 3 we discuss the symmetry structure, operator content, and target space geometry of the quiver Yang-Mills theories (with particularly symmetric choices for the parameters of the gauge theories) and compare them to those of the corresponding orbifold CFT. The agreement between the two provides strong evidence that the quiver theories flow (upon appropriate fine-tuning) to the corresponding orbifold CFT in the infrared, as we explain in section 3.3; we discuss interesting applications of this fact in section 3.4. In section 4 we present calculations which show that fine-tuning is in fact required at subleading orders in the $1/N$ expansion of the non-supersymmetric quiver theories; we constrast this to the case of D-branes at asymmetric orbifolds where such fine-tuning is not required at the order we compute. Finally, in section 5 we discuss a simple example of a non-supersymmetric string background in which the worldsheet CFT becomes singular and consider the worldvolume theories on D-branes in such backgrounds.

2. The theories

In this section we present the theories we will study. Most of our analysis will involve the symmetric orbifold theories in section 2.1, which we will compare and contrast with analogous results for the theories in section 2.2 and section 2.3.

2.1 Quiver theory

The worldvolume theories on Dp-branes at symmetric orbifold singularities were worked out in [10, 9]. In general, one can consider an orbifold $\mathbb{C}^4/\Gamma$ where Γ is a subgroup of the Poincare group of $\mathbb{C}^4$. The orbifold group acts on the Chan-Paton factors on the open string endpoints in addition to acting on the Lorentz quantum numbers of the fields. In the case that $\Gamma = \mathbb{Z}_k$, one finds a gauge group $\mathrm{U}(N)^k$ and matter in various bifundamental representations determined by the details of the geometrical action of Γ. If the Chan-Paton factors sit in a regular representation of Γ, then the quiver theory describes Dp-branes that can all move off the orbifold fixed point. In the case of a non-regular representation, there are fractional branes stuck at the fixed point, which at least in some cases have been seen to be equivalent to higher dimensional D-branes wrapped around a collapsed cycle implicit in the singularity.

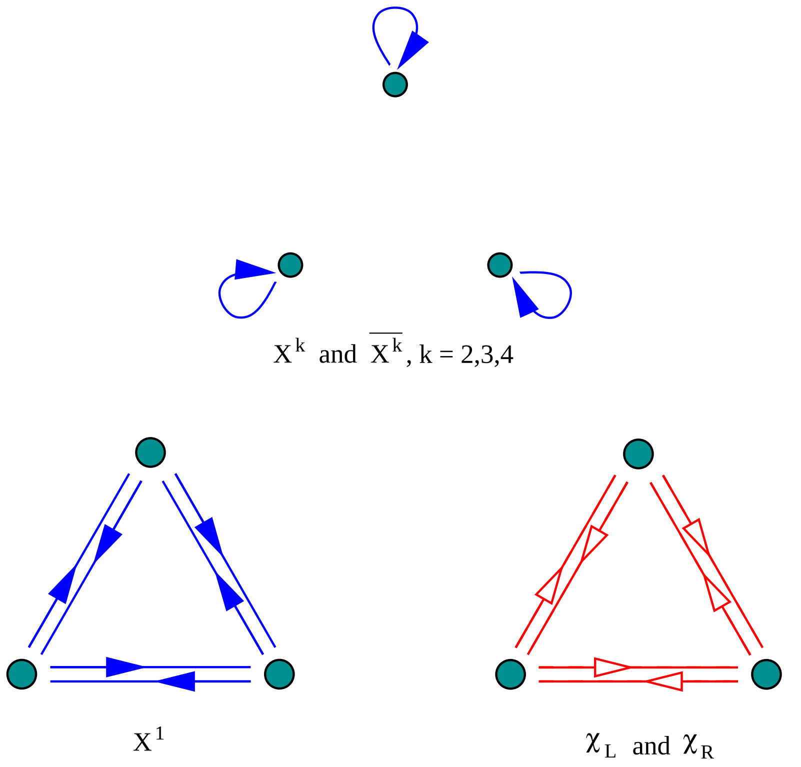

Figure 1: $\mathbb{C}/\mathbb{Z}_3$ model quiver diagram.

We will consider the following supersymmetry-breaking model in some detail. We will take a D1-brane transverse to $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{C}^3$. The $\mathbb{Z}_3$ acts on the four complex coordinates (x^1, x^2, x^3, x^4) transverse to the D1-brane by

$$(x^1, x^2, x^3, x^4) \to (e^{2\pi i(2/3)}x^1, x^2, x^3, x^4)\,. \tag{2.1}$$

The unorbifolded D1-brane has a gauge group $\mathrm{U}(3N)$ with fermions and scalars in the adjoint representation. Left-moving fermions $\chi_L^\alpha, \alpha = 1, \ldots, 8$ transform in the $\mathbf{8_s}$ of the SO(8) rotation group transverse to the branes, and right-moving fermions $\chi_R^{\dot\alpha}, \dot\alpha = 1, \ldots, 8$ transform in the $\mathbf{8_c}$ of that SO(8). Breaking up the eight real left-moving fermions into four complex fermions, one finds that the action (2.1) rotates them by $e^{2\pi i/3}$, and similarly for the right-movers. This means in particular that the orbifold breaks all the supersymmetry. The action on the scalars (X^1, X^2, X^3, X^4), whose eigenvalues parameterize the position of the brane on the transverse $\mathbb{C}^4$, is as indicated in (2.1).

Projecting onto invariant states (taking the regular representation for the action on the Chan-Paton indices), on finds the following spectrum. The gauge group is $\mathrm{U}(N)^3$. The components of X^2, X^3, and X^4 that are invariant under the orbifold transform in the adjoint of $\mathrm{U}(N)^3$. X^1 and the fermions χ_L, χ_R transform in the $(N, \overline{N}, 1)+(1, N, \overline{N})+(\overline{N}, 1, N)$ representation. The quiver diagram for this theory [9] is shown in figure 1.

The interactions of the theory are those of the unorbifolded $\mathcal{N} = 8$ theory which involve worldvolume fields invariant under the orbifold. The diagonal U(1) in the $\mathrm{U}(N)^3$ gauge group is totally decoupled (nothing is charged under it). There are two "relative" U(1) factors which couple. The lagrangian includes theta-terms $\theta_i \int F_i$ where F_i are the field strengths of the three U(1) factors. We will analyze the symmetry structure of the theory arising from these interactions (choosing particularly symmetric values for the quiver theory couplings) in studying their renormalization-group flow in section 3.

2.2 D1-D5

We will also briefly consider supersymmetry-breaking theories involving both D1-branes and D5-branes. Let us consider a $\mathbb{C}/\mathbb{Z}_3$ orbifold transverse to the D1-D5 system, taking the regular representation for the action of the orbifold group on both the D1-branes and the D5-branes. (Otherwise the (1-5) and (5-1)-sector strings would be projected out by the orbifold action.) This involves considering 3 sets of Q_1 D1-branes and 3 sets of Q_5 D5-branes. Before orbifolding, the (1-1) sector strings are as described in the previous subsection, and the (1-5) and (5-1) strings form hypermultiplets in the $(\mathbf{3Q_1}, \mathbf{3Q_5})$ representation of the $\mathrm{U}(3Q_1) \times \mathrm{U}(3Q_5)$ symmetry group of the D1-branes and D5-branes. After orbifolding, we obtain figure 2.

2.3 Asymmetric orbifolds

We will find in some sense more positive results for the case of D-branes on certain asymmetric orbifold backgrounds [16]. In this subsection we briefly review what we will need about their worldvolume theories [22]. Consider an asymmetric orbifold involving an action by $(-1,1)^4(-1)^{F_R}$ (that is, a $\mathbb{Z}_2$ reflection of the left-moving coordinates and no action on the right-moving coordinates on the string worldsheet). This is a symmetry of T^4 compactification at the SO(8)-symmetric point in Narain moduli space, and is a basic ingredient in the models considered in [16].

This action maps D1-branes transverse to the T^4 into D5-branes wrapped on it, and therefore is only a symmetry if we take $Q_1 = Q_5$ in the unorbifolded theory. The orbifold action then acts off-diagonally on (a natural basis of) the open-string Hilbert space. Strings from the (1-1) sector map to (5-5) strings, (1-5) to (5-1). This means that if we start with a (1-1) sector string state $|\phi\rangle_{1-1}$, it will automatically combine with a (5-5) string state $|g\phi\rangle_{5-5}$ (where $g\phi$ denotes the result of acting with the orbifold group on the worldsheet modes ϕ) to form an invariant linear combination $|\phi\rangle_{1-1} + |g\phi\rangle_{5-5}$. This will be significant later.

3. General features

In this section we will assemble information on the target space, symmetries, and operator content of the quiver gauge theories. This will provide strong evidence for a (possibly fine-tuned) flow from the quiver Yang-Mills theories for particular values of

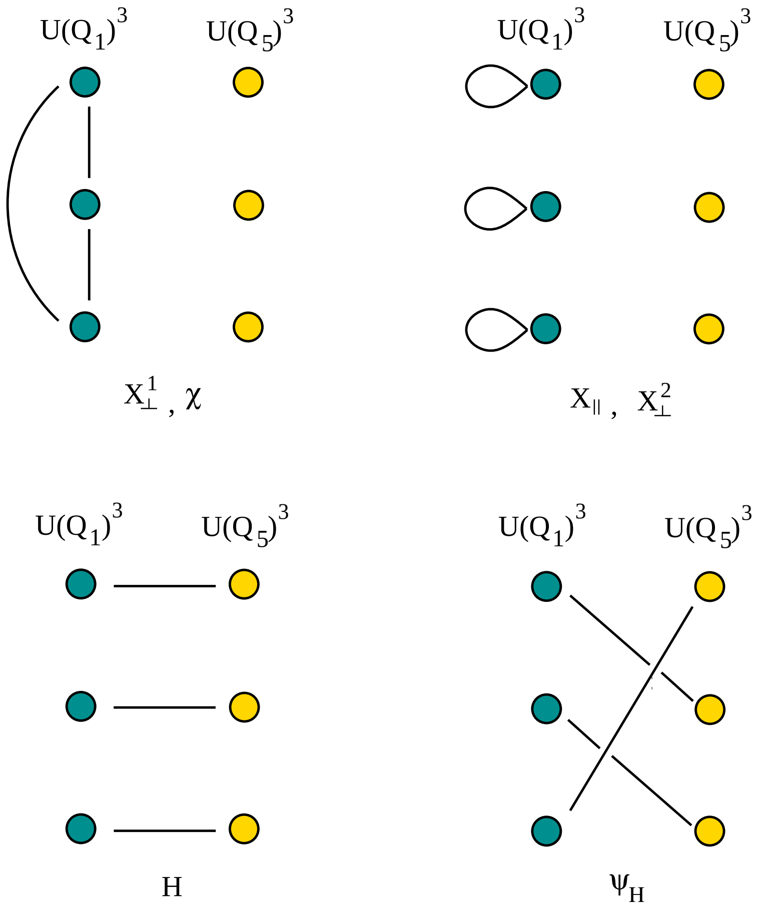

Figure 2: D1-D5 quiver diagrams. We schematically denote conjugate pairs by a single line, and therefore omit arrows in the diagram. $X_\perp$ are the coordinates perpendicular to the D1-D5 brane system, whereas $X_\parallel$ denote the position of D1-branes on D5-branes. H and ψ_H are hypermultiplet fields coming from the (1-5) and (5-1) strings.

the parameters to orbifold conformal field theory in the infrared. In particular in section 3.1 we will determine the appropriate values of the θ-parameters corresponding to the orbifold conformal field theory.

3.1 Interactions and symmetries

We are interested in whether the quiver gauge theory obtained on D1-branes on an orbifold background flows to the corresponding orbifold CFT in the infrared. A $\mathbb{Z}_k$ orbifold CFT has two types of discrete symmetries. The first is an exchange of the g-twisted sector with the g^{-1}-twisted sector (for all orbifold group elements g) combined with left$\leftrightarrow$right exchange (parity). Let us call this transformation "t-Parity". The second is the $\mathbb{Z}_k$ quantum symmetry [23] which constrains the correlators of twisted sector vertex operators. Also, the orbifold conformal field theory is nonsingular. In supersymmetric contexts, this requires that the UV Yang-Mills theory not have a Coulomb branch.

For simplicity we will here consider $\mathbb{Z}_3$ orbifolds such as the $\mathbb{C}/\mathbb{Z}_3$ case that is our main example, but the results of this subsection are more general (and in particular apply to the supersymmetric cases of [18, 17]). If we choose the gauge couplings and matter self-interactions as inherited from the unorbifolded theory, then these terms respect t-parity and a $\mathbb{Z}_3$ symmetry under which the three $\mathrm{U}(N)$ factors in the gauge group are cyclically permuted.

Let us consider the θ-parameters in the field theory. In general these couple via the terms

$$\mathcal{L}_\theta = \theta_1 \int F_1 + \theta_2 \int F_2 + \theta_3 \int F_3 \,. \tag{3.1}$$

This can be rewritten as (writing $\alpha = e^{2\pi i/3}$)

$$\begin{aligned}\mathcal{L}_\theta = \int & \left(\frac{\theta_1 + \alpha^{-1}\theta_2 + \alpha\theta_3}{3}\right)(F_1 + \alpha F_2 + \alpha^{-1} F_3) + \\ & + \left(\frac{\theta_1 + \alpha\theta_2 + \alpha^{-1}\theta_3}{3}\right)(F_1 + \alpha^{-1} F_2 + \alpha F_3) + \\ & + \left(\frac{\theta_1 + \theta_2 + \theta_3}{3}\right)(F_1 + F_2 + F_3)\,. \end{aligned} \tag{3.2}$$

This basis is useful for studying the possible correspondence with orbifold conformal field theory because each term here is an eigenstate of the $\mathbb{Z}_3$ symmetry which permutes the three $\mathrm{U}(1)$ factors. Let us set

$$\begin{aligned}\eta &= \frac{\theta_1 + \alpha^{-1}\theta_2 + \alpha\theta_3}{3} \\ \bar\eta &= \frac{\theta_1 + \alpha\theta_2 + \alpha^{-1}\theta_3}{3}\,. \end{aligned} \tag{3.3}$$

The theory is invariant under the shifts $\theta_i \to \theta + 2\pi n_i$. for integer n_i. In terms of the complex parameter η, this is the equivalence

$$\eta \cong \eta + \frac{2\pi}{3}(n_1 + \alpha^{-1} n_2 + \alpha n_3). \tag{3.4}$$

We would like to find the values η can take such that:

(i) there is a $\mathbb{Z}_3$ symmetry permuting the three $\mathrm{U}(1)$ factors,

(ii) there is a t-Parity symmetry, and

(iii) the model has no Coulomb branch.

Translating these conditions into conditions on η, they become

$$\begin{aligned} &\text{(i)} \quad \eta + \frac{2\pi}{3}(m_1 + \alpha^{-1} m_2 + \alpha m_3) = \alpha\eta \\ &\text{(ii)} \quad \eta + \frac{2\pi}{3}(n_1 + \alpha^{-1} n_2 + \alpha n_3) = -\bar\eta \\ &\text{(iii)} \quad \eta \ncong 0\,. \end{aligned} \tag{3.5}$$

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The condition (iii) must be imposed since it ensures that there is a nonzero background electric field which contributes positive vacuum energy obstructing the Coulomb branch, as explained for this type of model in [8].

These conditions are solved by

$$\eta = \frac{1}{3}\left[-\pi + i\frac{\sqrt{3}}{2}\left(\frac{2\pi}{3}\right)\right]. \tag{3.6}$$

Another simple basis to work in is the following.

$$\mathcal{L}_\theta = \int \left(\frac{2\theta_1 - \theta_2 - \theta_3}{3}\right)(F_1 - F_2) + \left(\frac{2\theta_3 - \theta_1 - \theta_2}{3}\right)(F_3 - F_2) + \theta_{\rm tot} F_{\rm tot}\,, \tag{3.7}$$

where $\theta_{\rm tot}F_{\rm tot} = (\frac{\theta_1+\theta_2+\theta_3}{3})(F_1 + F_2 + F_3)$. Let us set $\kappa_{12} \equiv \frac{2\theta_1-\theta_2-\theta_3}{3}$ and $\kappa_{32} \equiv \frac{2\theta_3-\theta_1-\theta_2}{3}$.

In this basis, the solution (3.6) for η is

$$(\kappa_{12}, \kappa_{32}) = \left(-\frac{2\pi}{3}, \frac{2\pi}{3}\right). \tag{3.8}$$

This agrees with the results of [17]. This procedure of enforcing t-Parity, the quantum symmetry, and the absence of a Coulomb branch of course applies much more generally than just to the $\mathbb{Z}_3$ orbifold of interest here.

We can also consider continuous global symmetries in addition to the discrete symmetries we have so far considered. Chiral symmetries in particular can provide exact, non-perturbative constraints on renormalization group flow. As proposed by 't Hooft [24], anomaly coefficients remain invariant under the renormalization-group flow, and hence their matching can serve as a useful consistency check. In our $\mathbb{C}/\mathbb{Z}_3$ example, however, we find by studying the surviving interactions in the theory that there are no chiral symmetries in the UV with nontrivial 't Hooft anomalies. This was noted in the case of D1-branes in flat space in [25]. In quiver theories with for example (2,2) supersymmetry, one does find chiral symmetries, but they act on scalars which parameterize the target space of the D1-brane worldvolume theory. These symmetries therefore do not match on to the R-symmetries in the affine Lie algebra of the infrared superconformal field theory. In our supersymmetry-breaking example, we do not find even this type of chiral symmetry. We therefore cannot use the 't Hooft anomaly matching condition to constrain the flow. The absence of such chiral symmetries has implications for matrix string theory as we discuss further below.

3.2 Moduli space

Let us consider the $\mathbb{C}/\mathbb{Z}_3$ theory of section 2.1. The classical moduli space of the ultraviolet Yang-Mills theory on the D1-branes has two branches. One of them, the Higgs branch, is simply the orbifold space

$$\mathcal{M}^{\rm class}_{\rm Higgs} = (\mathbb{C}/\mathbb{Z}_3 \times \mathbb{R}^6)^N / S_N\,. \tag{3.9}$$

This is true simply by construction; this branch describes the N branes moving around on the orbifold space we started with.

If the branes coalesce at the singularity, they can separate in the $\mathbb{R}^6$ transverse to the orbifolded dimensions. This yields a classical Coulomb branch of the form

$$\mathcal{M}^{\text{class}}_{\text{Coulomb}} = (\mathbb{R}^6)^{3N}/S_{3N} \,. \tag{3.10}$$

This branch only exists for $\kappa_{12} = 0$, $\kappa_{32} = 0$.

3.3 Operator spectrum and the flow conjecture

Let us summarize the situation so far. If we consider the theory at the values of the θ parameters determined in section 3.1, then the theory has several features which will be of interest:

1. An unbroken t-Parity symmetry, and no Coulomb branch.
2. The Higgs branch (3.9) is a simple orbifold space.
3. There is also a $\mathbb{Z}_3$ discrete symmetry in the model, acting on operators which are gauge-invariant in the orbifold but would not have been in the original theory.

These features (1)–(3) are also true of the orbifold conformal field theory on $(\mathbb{C}/\mathbb{Z}_3 \times \mathbb{R}^6)^N/S_N$:

1. This is a (nonsingular) symmetric orbifold conformal field theory, so there is a left-right exchange symmetry combined with exchange of congugate twisted sectors (t-Parity).
2. The target space is this orbifold space.
3. The orbifold CFT includes twisted sectors that transform under a discrete $\mathbb{Z}_3$ "quantum" symmetry [23] that constrains their correlators.

These features suggest that the ultraviolet Yang-Mills theory might flow in the infrared to the corresponding orbifold conformal field theory. Let us consider the operator spectrum of the two theories.

The orbifold CFT has relevant operators of the form

$$\begin{aligned} &e^{ikY} + (\text{images under } \mathbb{Z}_3 \text{ and } S_N \text{ actions}) \\ &\frac{1}{2}k^2 < 1 \,, \end{aligned} \tag{3.11}$$

where Y is a coordinate in $(\mathbb{C} \times \mathbb{R}^6)^N$. If we consider a compact target space then there is a finite number of such operators. These operators are not charged under any symmetry of the theory. If we add them to the lagrangian they appear as contributions to the scalar potential. Being relevant, adding them to the lagrangian drives the theory away from the orbifold CFT fixed point.

In the ultraviolet Yang-Mills theory, there is correspondingly a scalar potential $V(Y^I)$, where Y^I are gauge-invariant operators parameterizing the classical moduli space. In the model inherited from the orbifold in string theory, this potential is zero classically. (In the quantum field theory one could consider any $V(Y^I)$ classically, and we will use this freedom shortly.) There is an infinite number of possible terms in $V(Y^I)$ that are not charged under any symmetry of the orbifold theory, which therefore could be generated under renormalization group flow. Since for large $|Y|$ the D1-branes separate and only feel the supersymmetry-breaking through stretched strings of mass proportional to $|Y|$, the potential should fall to zero as $|Y| \to \infty$.[1] In the next section, we will explicitly calculate quantum corrections in the Yang-Mills regime and find that a nontrivial potential $V(Y^I)$ is in fact generated at subleading orders in a $1/N$ expansion.

From (3.11) we see that only a subset of these terms become relevant operators from the point of view of the IR orbifold conformal field theory. It is therefore possible to adjust parameters in the UV Yang-Mills lagrangian to cancel off the contributions that will become relevant in the infrared. In the case of the noncompact target space (3.9), this set of relevant perturbations, though a subset of the perturbations that can be considered in the Yang-Mills theory, is still a continuous infinity of perturbations, and the fine-tuning we are discussing would need to be applied to an infinite number of coefficients. This can be avoided by compactifying the target space.

One could also generate corrections to the metric on the target space (3.9). In the orbifold CFT such corrections, which can be Fourier-expanded to the form $\partial Y \partial Y e^{ikY}$, are all irrelevant. Therefore these terms do not need to be fine-tuned away in order to flow to the orbifold conformal field theory.

In the orbifold CFT, there are relevant operators in the $\mathbb{Z}_3$ twisted sectors. These are charged under the $\mathbb{Z}_3$ quantum symmetry of the orbifold, which, as mentioned above, exists also in the UV Yang-Mills theory for η given by (3.6). In the UV theory the operators charged under the $\mathbb{Z}_3$ symmetry cannot be generated under RG flow.

There are other symmetries of the orbifold CFT that are not mirrored in the gauge theory. The orbifold CFT has a (2,2) superconformal algebra. This algebra includes left and right-moving $\mathrm{U}(1)_R$ symmetries under which some operators (twist fields) have fractional charges, so that there is no spacetime supersymmetry. There is no such chiral symmetry in the Yang-Mills quiver theories, as we discussed in section 3.1.

[1]One finds that logarithmic terms do not arise because of cancellations in the planar diagrams reviewed below.

Despite the appearance of an "accidental" supersymmetry algebra in the IR which has no counterpart in the UV Yang-Mills theory, there are many features that do agree between the two theories. The identification of the targets space, discrete symmetries, and untwisted relevant perturbations suggests strongly that the quiver Yang-Mills theory is fine-tunable to flow to the corresponding orbifold CFT in the infrared.

3.4 Applications to string theory and gravity

Given a well-defined (if fine-tuned) flow from the Yang-Mills theory to the orbifold CFT, it is intriguing to consider whether it defines an interesting gravity dual. At least naively, one can fine-tune the boundary conditions in (an orbifold of) the gravity solution of [3] to obtain an ultraviolet lagrangian with the right values of the couplings to flow to the orbifold CFT. It will be interesting to try to work this out. Similarly we can consider the question of whether this flow defines a matrix string theory which formulates a spacetime string theory away from its $g_s \to 0$ limit.

Another interesting application is the following. If we consider the $N = 1$ theory, the orbifold CFT lives simply on $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{R}^6$, and constitutes a CFT that could appear on the worldsheet of a perturbative string. In this context, the relevant operator in the twisted sector is a vertex operator for a twisted-sector tachyon in spacetime. This is a twist field, an operator nonlocal with respect to the elementary degrees of freedom of the orbifold sigma model. In the UV Yang-Mills theory, the corresponding operator (whatever it is) is a simple gauge-invariant combination of elementary fields in the Yang-Mills theory. Examples of "twisted" operators in the Yang-Mills theory include relative gauge couplings

$$\mathrm{Tr}\, F_1^2 - \mathrm{Tr}\, F_2^2 \tag{3.12}$$

and for example couplings of the form

$$a \left| X^1_{(+-0)} \right|^2 + b \left| X^1_{(0+-)} \right|^2 \tag{3.13}$$

(analogues of relative D terms in supersymmetric theories). Here the subscripts denote the components of X^1 involved via their charges under the $\mathrm{U}(1)^3$ gauge symmetry. It would be very nice to identify the operator that corresponds to the vertex operators for the twisted sector tachyons, since the effect of adding them to the lagrangian might be clearer in the Yang-Mills theory. In particular, we might get information on how much central charge is lost upon turning on the relevant deformation corresponding to tachyon condensation [26]; this type of tachyon (localized at fixed points of an orbifold) was not covered by the techniques in [27], where other types of closed string tachyons were analyzed.

The absence of the $\mathrm{U}(1)_R$ chiral symmetry in the UV theory has implications for matrix string theory (with any amount of supersymmetry). One might have hoped to find that the worldsheet supersymmetry of the infrared $g_s \to 0$ conformal field theory would extend to the full $g_s \neq 0$ matrix string theory. In the infrared this symmetry is made manifest by bosonizing the Green-Schwarz variables and re-fermionizing them to obtain the RNS description of the superstring where worldsheet supersymmetry is apparent. Given a chiral U(1) current in the UV with the right properties to match onto the $\mathrm{U}(1)_R$ symmetry of the IR superconformal algebra, one might have been able to bosonize that current away from criticality and find some sort of RNS formulation of the matrix string theory. This does not seem possible for the quiver theories (or the parent theory of D1-branes in flat space); it would be interesting to find backgrounds for which such chiral symmetries *would* appear on the worldvolume of D1-branes.

4. Diagrammatics

In this section we perform explicit calculations to check whether the terms of the form $V(|Y|)$ are in fact generated under RG flow in the Yang-Mills regime (g_{YM} small) of the quiver theory. After reviewing the fact that they are not generated by planar diagrams, we show that they are generated by nonplanar diagrams. By way of contrast, we show that in the asymmetric orbifold case of section 2.3, such terms are *not* generated at the order we compute. For simplicity in this section we will work with the $N = 1$ theory of a single D1-brane on the orbifold background.

4.1 Planar diagrams

The quiver quantum field theory has a dual description in terms of an orbifold of the gravity solution of [3], which acts on the sphere surrounding the branes but not on the radial coordinate [5]. In the worldsheet string theory describing this gravity background, correlation functions of untwisted vertex operators are inherited from those of the unorbifolded theory. This ensures that the correlation functions of untwisted operators in the gauge theory are similarly inherited, if the correspondence is correct (as explained for the β-functions in [5]). This result can be seen directly to all orders in perturbation theory by an analysis of the field theory diagrams [12].

So for example in the case of the 1-loop contributions to the scalar mass squared for X^1 in the $\mathbb{C}/\mathbb{Z}_3$ quiver theory, one finds the graphs of figure 3.

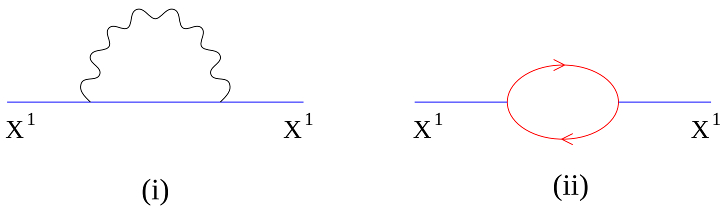

Figure 3: Planar 1-loop Feynman diagrams contributing to the X^1 mass term in the $\mathbb{C}/\mathbb{Z}_3$ quiver theory.

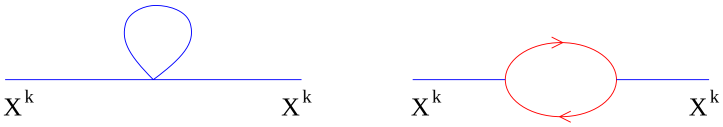

Figure 4: 1-loop contribution to the scalar potential on the Coulomb branch. The index $k = 2, 3$ or 4.

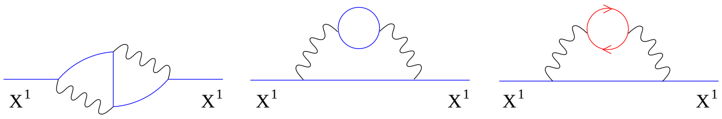

Figure 5: 2-loop contributions to the scalar potential on the Higgs branch.

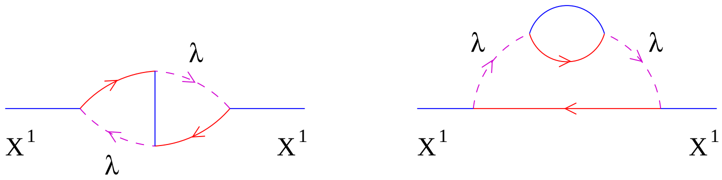

Figure 6: 2-loop "superpartner" Feynman diagrams whose contributions would cancel against those in figure 5. Here λ denotes gauginos.

The diagram (i) with bosons running in the loop cancels against a diagram (ii) with fermions running in the loop; although the fermions are not superpartners of the bosons they come in the right multiplicities to cancel and the Lorentz structure of the diagrams are the same as appears in the supersymmetric theory.

4.2 Non-planar diagrams

Let us first consider non-planar contributions to the scalar potential on the Coulomb branch. Let us calculate the mass squared of the modes describing the relative motion of the branes in the $X^{2,3,4}$ directions. At one-loop the relevant diagrams are in figure 4.

This includes nonplanar contributions. The zero-external-momentum piece of this diagram does not cancel. This can be seen from the fact that there is an excess of fermions running in the loop as compared to the balance of fermions and bosons in corresponding diagrams in a supersymmetric theory.

On the Higgs branch the relevant quantity to compute is the scalar potential for X^1. Here there are no non-planar contributions at one loop. At two loops we find nonplanar diagrams of the form given in figure 5.

In a supersymmetric theory, their contribution to the scalar potential would cancel against the contribution of the following graphs in figure 6 involving the superpartners. These fermions running in the loops of these diagrams include adjoint fermions (gauginos in a supersymmetric theory). Evaluating these "superpartner"

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diagrams, we find for the first one

$$
\begin{aligned}
(-ig_{YM})^4 \int d^2q d^2k \frac{tr((\not{k}+m)\not{q}(\not{q}+m)\not{k})}{(q^2+m^2)q^2(k^2+m^2)k^2(q-k)^2} &= \\
= (-ig_{YM})^4 \int d^2q d^2k \frac{k^2q^2+m^2q\cdot k}{(q^2+m^2)q^2(k^2+m^2)k^2(q-k)^2} &. \qquad (4.1)
\end{aligned}
$$

For the second one we find

$$
\begin{aligned}
(-ig_{YM})^4 \int d^2q d^2k \frac{tr((\not{k}+m)\not{q}(\not{k}+m)\not{k})}{q^2(k^2+m^2)^2k^2(q-k)^2} &= \\
= (-ig_{YM})^4 \int d^2q d^2k \frac{(k\cdot q)(k^2+m^2)}{(k^2+m^2)^2k^2q^2(q-k)^2} &. \qquad (4.2)
\end{aligned}
$$

Here the mass m for the gauginos is proportional to $g_{YM}\rho$, where ρ is the distance of the D1-brane from the orbifold fixed point. Let us consider the integrands of these diagrams. We find, upon introducing Feynman parameters, an expression that separates into a piece that integrates to zero plus a piece whose integrand is of definite sign that cannot cancel. To begin with the $k\cdot q$ terms in (4.1) and (4.2) are of indefinite sign. Introducing Feynman parameters, one obtains an integrand of the form

$$
\left(\frac{f(k^2,m^2)}{D((q')^2,k^2)}\right)^5 k\cdot(q'+ak)\,, \qquad (4.3)
$$

for each of these contributions. Here the denominator D is a positive (indefinite) function of $(q')^2$ and k^2, where q' is equal to $q-ak$ for a positive quantity a (which is a function of Feynman parameters); $f(k^2,m^2)$ is a positive function of k^2 and m^2. Given this, the $k\cdot q'$ term in the numerator integrates to zero since the integrand is odd in q' and k. The other term ak^2 has a positive integrand. These surviving contributions to the integrand from the $k\cdot q$ terms have the same sign as the other terms in (4.1) and (4.2).

Since the surviving integrand here is of definite sign, the contribution of the "superpartner" graphs is nonzero, independent of the precise infrared regularization scheme that we might introduce, as long as that scheme respects Lorentz invariance. (So this result would hold for example if we introduce masses to regularize the infrared divergences.)

This above non-cancellation of $1/N$ corrections to the D1 worldvolume theory will occur for generic nonsupersymmetric quiver theories. The simplest way to see this is to note that the absence of adjoint fermions (which would run in the loops of (4.1) and (4.2)) is a direct consequence of the fact that the orbifold action projects out all spacetime spinors. Given a nontrivial action on the spinor quantum numbers of the worldvolume fermions, the surviving representations are necessarily bifundamental rather than adjoint.

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4.3 Asymmetric orbifolds

This is not the case for the asymmetric orbifolds reviewed briefly in section 2.3. For these theories, the invariant states of the form $|\phi\rangle_{1-1} + |g\phi\rangle_{5-5}$ include adjoint fermions. Their interactions are also such that the "superpartner" diagrams (4.1) and (4.2) are present in the theory, in addition to the bosonic diagrams that they cancel. As such, at this order in the asymmetric orbifold background, the moduli space on the D-brane worldvolume is stable under quantum corrections. These diagrams are suppressed by a factor of $1/N^2$ relative to the leading order contributions. This cancellation is in accord with the 1-loop cancellation of the cosmological constant in this background [16] through AdS/CFT duality as explained in [5]. It would be very interesting to see where the cancellations break down (if at all) from the point of view of the D-brane worldvolume theories. In particular, the spacetime theory's 2-loop cancellation in the cosmological constant (according to a rather subtle calculation in [16]) suggests that these cancellations might persist at least to order $1/N^4$ in the D-brane theories.[2]

5. Singular CFTs

As discussed above, for $\theta = 0$ a new (Coulomb) branch opens up in the target space of the Yang-Mills theory. In supersymmetric situations, the existence of this branch is stable against quantum corrections (although its geometry is subject to corrections). As first explained in the context of Calabi-Yau compactification in [8], these branches lead to singularities in correlation functions arising from integration over the bosonic zero modes parameterizing this branch [30]. In the application to worldsheet string theory, these singularities in the infrared CFT describe spacetime backgrounds of string theory in which light non-perturbative states appear (and resolve the singularities in the correlation functions). In particular in compactifications of type II string theory to six dimensions on $K3$, one finds non-perturbatively enhanced non-abelian gauge symmetry at points in moduli space where the geometry is an orbifold space (an ADE singularity) and the integrals $\int_{C_i} B$ of the NS 2-form B over the collapsed 2-cycles C_i of the ADE singularity vanish [19]. The quantum-mechanical explanation of the singularity is different in different supersymmetry classes [31]. It is interesting to consider whether this kind of phenomenon arises also in non-supersymmetric string backgrounds.

[2]This two-loop cancellation is based on the rather subtle formalism of integration over split RNS supermoduli space at genus two, evaluated in a particular gauge with separate analysis of the cancellation of boundary contributions. A recent work [28] objecting to this conclusion for a subset of these theories includes a not-yet-complete calculation in a different gauge, and (as far as we understand) an invalid objection to a determinant factor in the measure of the integral over supermoduli space included in the calculation of [16]. Independent tests of the result of the calculation in [16] (and therefore of the formalism involved) will involve integration over the moduli space of the diagram; such tests might be most tractable near boundaries of the moduli space [29].

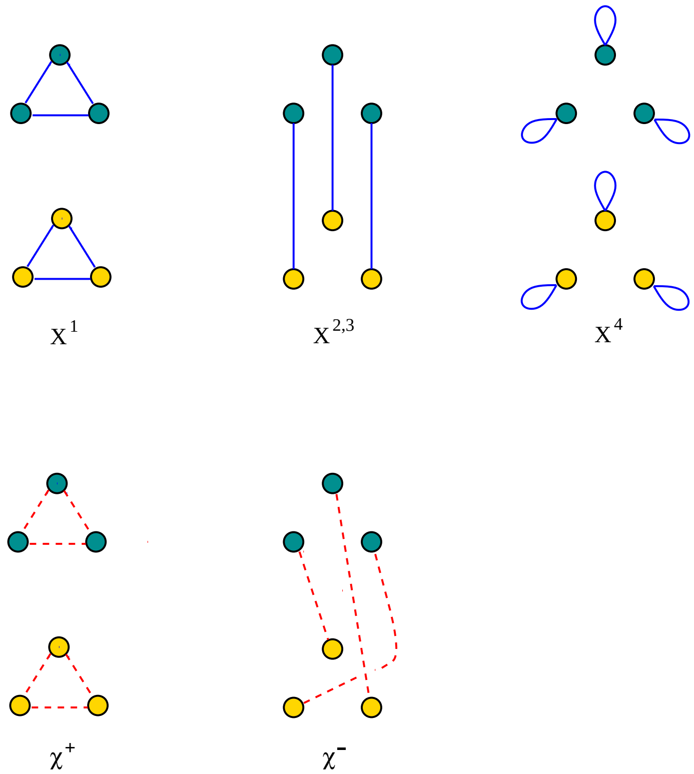

Figure 7: $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{C}^2/\mathbb{Z}_2$ quiver Diagrams. χ^+ (χ^-) collectively denote the spinors with $+1(-1)$ eigenvalues under the $\mathbb{Z}_2$ action on $\mathbb{C}^2$.

In generic non-supersymmetric orbifold conformal field theory (for example on the $\mathbb{C}/\mathbb{Z}_3$ background we have been considering) one does not expect exactly marginal operators like $\int B$ of the above example. Let us instead consider an orbifold conformal field theory on the product space $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{C}^2/\mathbb{Z}_2$. The second factor alone preserves supersymmetry and does have exactly marginal operator in the $\mathbb{Z}_2$ twisted sector corresponding to $\int B$. This exactly marginal operator is trivially present in the product as well, and when $\int B \to 0$ the correlation functions of the full theory (in particular those involving $\mathbb{Z}_3$-invariant, $\mathbb{Z}_3$-untwisted operators) diverge as in the supersymmetric theory.

This singularity in the non-supersymmetric worldsheet conformal field theory as $\int B \to 0$ will not cancel upon considering condensation of the $\mathbb{Z}_3$-sector tachyon and higher-loop effects. It suggests the presence of light states in the theory not described by fundamental strings. It would be interesting to understand the resolution of this singularity in the spacetime theory. Given such non-supersymmetric backgrounds not described by perturbative string theory, it would be interesting to understand the contributions these new light states make to quantum effects in these backgrounds.

We can construct the quiver theory corresponding to this product orbifold CFT. One finds a $\mathrm{U}(1)^6$ gauge group and matter content given by the quiver diagrams given in figure 7.

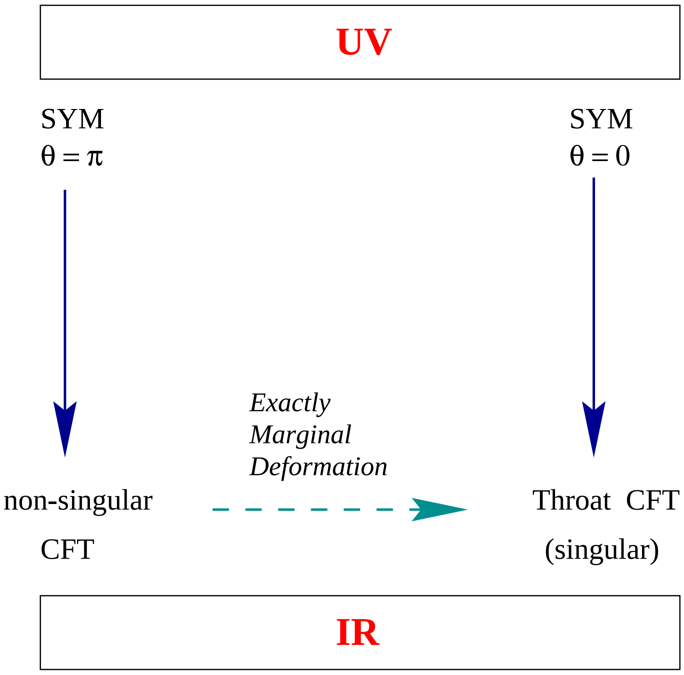

Figure 8: An expected flow pattern in the presence of supersymmetry.

Note that in the Yang-Mills theory, all degrees of freedom mix with each other and there is no product structure. In the quiver theory there is a branch of classical moduli space in which fractional branes separate at the $\mathbb{Z}_2$ fixed point, when the relative θ angle $\theta_{\mathbb{Z}_2}$ of the $\mathbb{Z}_2$ part of the quiver theory vanishes.

When the θ angles of the UV theory satisfy the discrete symmetry constraints of section 3.1, the theory can be fine-tuned to flow to the orbifold CFT on $\mathbb{C}/\mathbb{Z}_3 \times \mathbb{C}^2/\mathbb{Z}_2$. It would be interesting to understand whether this feature persists as we take $\theta_{\mathbb{Z}_2}$ to zero (in particular one would like to know how much fine-tuning is required as a function of $\theta_{\mathbb{Z}_2}$). If it persists, then one could study the singularity via the Coulomb branch of the Yang-Mills theory as in other cases.

In [21], the singular CFT in (4,4) supersymmetric theories was studied, and an effective description in terms of a WZW model times a linear dilaton was obtained far down the throat of the target space. As in the case of orbifold conformal field theory, this type of CFT exists (and is exactly solvable) with arbitrary amounts of supersymmetry. It would be interesting to find examples of theories with broken supersymmetry where this type of "throat" theory emerges.

Unlike in the case of orbifold CFT, the WZW times linear dilaton part of the singular CFT describes only a piece of the target space, which must be matched on to the rest of the target space in a manner which is consistent with conformal invariance. One might hope to access conformally invariant theories of this type by arriving at them via marginal deformation from an understood conformal field theory, such as orbifold conformal field theory. Indeed, in the supersymmetric context one finds a flow pattern of the form given in figure 8.

For example, in the D1-D5 system with $Q_1 = 1$ and $Q_5 = 2$, the Higgs branch moduli space is $\mathbb{R}^4/\mathbb{Z}_2 \times \mathbb{R}^4$. There are four exactly marginal perturbations involving the twist fields for the $\mathbb{Z}_2$ action. In the D1-D5-system with a transverse $\mathbb{C}/\mathbb{Z}_3$ orbifold action described in section 2.2 (for $Q_1 = 1, Q_5 = 2$), we find a Higgs branch moduli space

$$\mathcal{M} = (\mathbb{R}^4/\mathbb{Z}_2)^3 \times \mathbb{R}^4 \,. \tag{5.1}$$

Unfortunately in this case, we find that the $\mathbb{R}^4/\mathbb{Z}_2$ twist fields have dimension greater than 1 (and are therefore irrelevant). This arises because there are more fermions transforming under the relevant $\mathbb{Z}_2$ factor in the Yang-Mills gauge group than in the corresponding supersymmetric theory. So here we cannot deform toward a singular CFT using a marginal operator, since none presents itself. It would be very interesting to understand whether nonetheless the $\theta = 0$ Yang-Mills theory flows to a nontrivial (singular) CFT in the IR directly, perhaps with fine-tuning.

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